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Do Stock Returns Follow Normal Distribution? : Evidence from Bangladesh

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Abstract: This paper tries to investigate the normality of stock returns in Bangladesh. To test the normality of daily and monthly log returns, Jarque-Bera test, Shapiro-Wilk test and Anderson-Darling test have been used on three indices of Dhaka Stock Exchange (DSE) – DSEX, DS30 and DSES along with a graphical (Q-Q Plot) presentation. The paper covers a sampling period from January 2013 to March 2023. Daily returns on all three indices have been found to be deviated from normality by all the three tests. Non-normality once again has been found in two indices (DS30 and DSES). Only, distribution of monthly returns of DSEX index has been approximately normal. The overall results have implication for stock market efficiency. This implies that when assessing risk, investors should not rely on the assumption of normality of stock returns. As per the results, the efficiency of stock market is questionable both in the short run and long run.

Keywords: *Normality; Returns; Skewness; Kurtosis; Distribution.*

Introduction: Stock returns are important variables in finance and economics, and understanding their distributional properties is crucial for many applications. One such property is normality, which is a fundamental assumption in many statistical models. Analysts working on financial models assume stock returns to be normally distributed for simplicity [1], but the world is complex. In a simple world, there would have been no COVID-19. In the very early days of COVID-19, in just a week (March 11 to March 18, 2020), Dhaka Stock Exchange Index (DSEX) saw a decline of about 15%. Such disastrous events raise questions about the assumption of normality. This paper tries to test the normality of stock returns in Bangladesh. The concept of the normality of stock returns is based on the idea that, in a simple world of finance, the distribution of stock returns should follow a normal distribution. This means that most of the returns are clustered around the average return and extreme positive or negative returns are relatively rare [1]. The reason of testing the normality of stock returns in Bangladesh is to clearly understand whether the simplified assumption of normality holds in reality. If the normality assumption does not hold in stock returns in Bangladesh, then there will be more bad days in the investors' portfolio than expected when market performs poorly.

Researchers across time tested the normality of stock returns. Many researchers have rejected the assumption of normality. On the other hand, some researchers found evidence of normality.

There is a substantial body of empirical evidence against the normality of stock returns. Some studies have found evidence of fat tails, which means that the distribution has more extreme values than would be expected under a normal distribution. An analysis of price fluctuations in cotton futures revealed that the distribution exhibited heavy tails, challenging the notion of normality. [2]. That means extreme events occurred more often than expected in case of prices following normal distribution. Fama did a similar study and found stock returns had excess kurtosis [3]. It means that the distribution had fatter tails compared to that under a normal distribution. N. Strong challenged the traditional assumption that stock returns follow a normal distribution [4]. In that paper, he argued that a more accurate model for stock returns is a lognormal distribution, which better captures the empirical properties of asset prices. The distribution of stock returns has been leptokurtic with fat-tails and almost symmetrical shape [5-7]. Test of both the marginal and joint moments of asset returns revealed that stock returns are not normally distributed [8]. Normality might be a plausible assumption for monthly stock returns, but it is not valid for daily returns. [9]. Market returns across Latin American stock markets (Peru, Argentina, Brazil, Chile, Colombia, Mexico, and Venezuela) exhibited non-normality during the 2000-2007 period [10]. A similar study in Bangladesh during the period of 2002 – 2010 saw similar results [11]. A study on 65 stock market indices from 2013 to 2016 revealed that the distribution of daily and weekly returns of equity indices is not normal. [12]. One of the most recent studies have used code-generated programs to test normality of stock returns and found no evidence of normality [13]. Non-normality in stock returns is also evident in studies conducted on Bangladesh. The Assumption of normality is violated in daily, weekly and monthly return series [11, 14, 15].

Despite the substantial evidence against normality, there are also studies that have found evidence in favor of normality. For example, in some recent studies [16, 17], stock returns have been found compatible with the normal distribution. Even, in Bangladesh, during the COVID-19 period, stock returns have not been normally distributed [18].

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Existing literature on the normality of stock returns presents mixed results. A majority of studies suggests that stock returns do not follow normal distribution, while some studies have reported a degree of normality. However, a significant research gap lies in the limited number of studies that comprehensively examined the normality of stock returns by considering both daily and monthly data. The objective of this new paper is to bridge this gap by testing the normality of both daily and monthly stock returns. By doing so, it is expected to provide a comprehensive view on the distribution characteristics of stock returns. The potential contribution of this study to the existing literature is to reaffirm or challenge the prevailing knowledge on the normality of stock returns. It will also facilitate the understanding of potential variations in normality across time. This comprehensive approach could offer valuable insights for risk assessment, portfolio management, and investment decision-making, making it a noteworthy addition to the ongoing discourse in the field of finance.

Materials & Method: Many statistical concepts have been applied in this paper which are highly technical. The following section will cover all these technical aspects one by one.

- ***Variable of Interest:*** Log normal stock (Index) returns

$$return_{(t)} = ln\left(\frac{P_t}{P_{t-1}}\right)$$

Where, $return_t$ = Index return on day/month t;
P_t = Value of the index on day 't'
P_{t-1} = Value of the index on day 't-1'
ln = Natural log
According to [4], log returns tend to follow a normal distribution which is a precondition for using most statistical tests. Logarithm returns are both theoretically and empirically justified as a measure of returns for testing the weak form market efficiency. On a theoretical ground, logarithmic returns are easily tractable. Empirically, logarithmic returns tend to follow a normal distribution which is a precondition for applying many statistical analyses.

- ***Sampling Technique:*** Over the time, there have been many stock indices in DSE. For sample selection, randomly three indices have been selected.
- ***Sampling Scope:*** The paper uses three stock market indices of DSE:
  - Dhaka Stock Exchange Index (DSEX)
  - Blue Chip Index (DS30)
  - Shariah Index (DSES)
- ***Sampling Period:*** This study uses daily market returns (changes in value of three indices) data of the Dhaka Stock Exchange for the period of January 2013 to March 2023.
- ***Data Frequency:*** Daily and Monthly.
- ***Sample Size:*** Sample size for both daily and monthly returns is similar for DSEX and DS30 Index since both the indices were launched at the same time (January 27, 2013). However, DSES Index was launched on January 20, 2014. As a result, sample size of DSES index is one year less compared to DSEX and DS30 Index.

Table 1. Sample Size.

Indices	Sample Size	
	Daily	Monthly
DSEX	2423	121
DS30	2423	121
DSES	2192	109

- **Data Source:** The data of daily prices of three indices have been collected from the DSE website (dsebd.org) for the time period, January 2013 to March 2023.

Tests of Normality: Normality tests are commonly applied to financial data, including stock returns, to assess whether they follow a normal distribution. Here are some common tests of normality that are applied to stock returns:

- **Descriptive Statistics:** Descriptive statistics can be useful for assessing normality of stock returns. Descriptive statistics includes mean, median, mode, skewness kurtosis, standard deviation, range etc.
- **Q-Q Plot:** A Q-Q plot (quantile-quantile plot) is a graphical method used to compare the distribution of a sample of data to a theoretical normal distribution. In a Q-Q plot, the sample data is plotted on the y-axis and the expected values from a normal distribution are plotted on the x-axis.

- To create a Q-Q plot, the data are first sorted in ascending order. Next, the sorted data are transformed to their quantiles (i.e., the values that divide the data into equal-sized groups), which are then plotted against the corresponding quantiles of the theoretical distribution on a graph. The theoretical quantiles are usually calculated assuming a normal distribution, but other distributions can also be used.
- **Jarque-Bera (JB) Test:** This test is based on [19, 20] papers. It uses skewness and kurtosis of the data. It tests whether the sample data exhibit the level of skewness and kurtosis of a normal distribution. The null hypothesis of the test is normality. When a p-value is less than the significance level, assumption of normality can be rejected.
- **Shapiro-Wilk (SW) Test:** This test is based on [21]. It compares the observed data with data expected under a theoretical normal distribution. The null hypothesis assumes normality. When a p-value is less than the significance level, assumption of normality can be rejected.
- **Anderson-Darling (AD) Test:** This test is based on [22]. It is also a comparison of the observed data to what is expected under a specific distribution, such as the normal distribution. The test gives greater weights to the tails of the distribution. The null hypothesis assumes normality. When a p-value is less than the significance level, assumption of normality can be rejected.

Results and Discussion: This section of the paper includes the presentation of all the outcomes in the form of statistical data, graphs, tables, and figures.

- **Descriptive Statistics**

  Since 2013, the average daily return on three market indices has been approximately 0.02% with a standard deviation ranging from 0.80%-0.93%. All three return series tend to show slightly positive skewness. However, the kurtosis is significantly high for all three indices.

Table 2. Descriptive Statistics of Daily Returns.

	DSEX	*DS30*	*DSES*
Mean	0.0002	0.0002	0.0002
Standard Error	0.0002	0.0002	0.0002
Median	0.0002	0.0000	0.0002
Standard Deviation	0.0087	0.0093	0.0080
Kurtosis	11.53	9.75	16.73
Skewness	0.40	0.43	0.63
Range	0.1654	0.1608	0.1690
Minimum	-0.0674	-0.0639	-0.0724
Maximum	0.0980	0.0968	0.0966
Count	2423	2423	2192

The average monthly return on three market indices has been approximately within 0.30%-0.34% with a standard deviation ranging from 4.49%-5.19%. Return series tend to show slightly positive skewness. However, the kurtosis is significantly low for all three indices.

Table 3. Descriptive Statistics on Weekly Index Returns.

	DSEX	*DS30*	*DSES*
Mean	0.0034	0.0033	0.0030
Standard Error	0.0043	0.0047	0.0043
Median	0.0027	-0.0022	0.0042
Standard Deviation	0.0477	0.0519	0.0449
Kurtosis	0.64	1.24	1.78
Skewness	0.34	0.67	0.04
Range	0.2592	0.2938	0.2751
Minimum	-0.1128	-0.1146	-0.1266
Maximum	0.1465	0.1793	0.1485
Count	121	121	109

- **Histogram**

The normality of index returns has been graphical presented by histograms below.

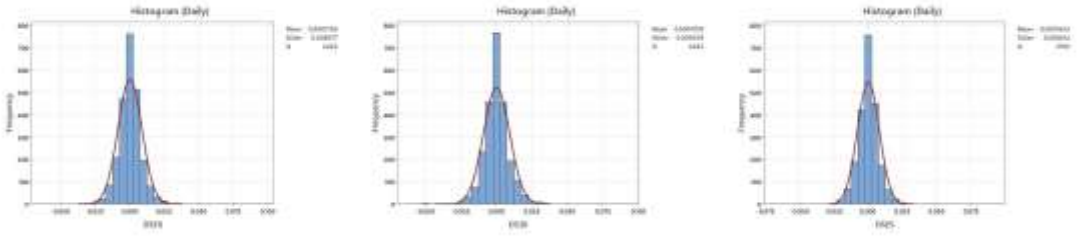

Figure 1 (a-c). Histogram of Daily Returns.

The histograms of daily returns in Figure 1 (a–c) above clearly show that all three indices are leptokurtic (excess kurtosis). However, skewness does not seem to be a problem for the distribution of any of the indices. In case of monthly returns in Figure 2 (a–c), the histograms presented below exhibit platykurtic (less kurtosis) distribution. Again, skewness does not seem to be a problem in monthly returns. So, overall, it is evident that the excess kurtosis of return series declines over time (from daily to monthly).

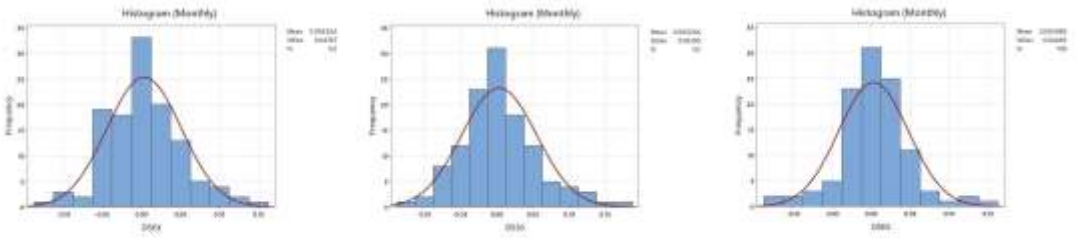

Figure 2 (a-c). Histogram of Monthly Returns.

- **Q-Q Plot**

The normality of index returns has been again graphical presented below, this time by using Q-Q Plots. Data are normally distributed when the points on the Q-Q plot fall on the theoretical straight line. If the data are deviated from the straight line, the data may not be normally distributed. Specifically, if the line is curved upwards, this suggests that the data are skewed to the right; if the line is curved downwards, this suggests that the data are skewed to the left; and if the line has kinks or bends, this suggests that the data have heavy tails.

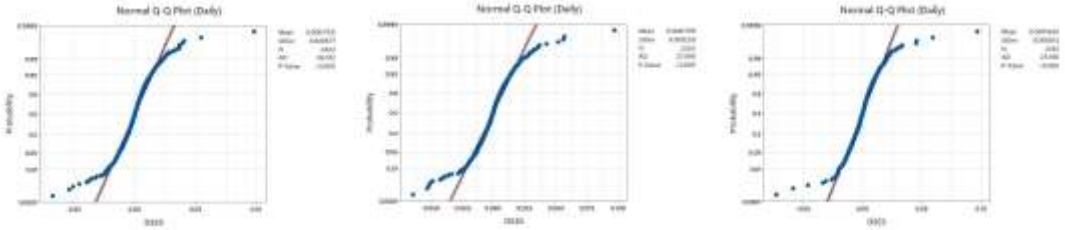

Figure 3 (a-c). Q-Q Plot of Daily Returns.

Q-Q Plots of daily returns in Figure 3 (a–c) of all three indices show that the line of actual data has kinks or bends. This suggests that the distributions have heavy tails. Heavy tails pose increased risk of rare, extreme events or black swan events, i.e., market crashes that can have catastrophic consequences. Traditional statistical models assuming normality, fail to predict these events.

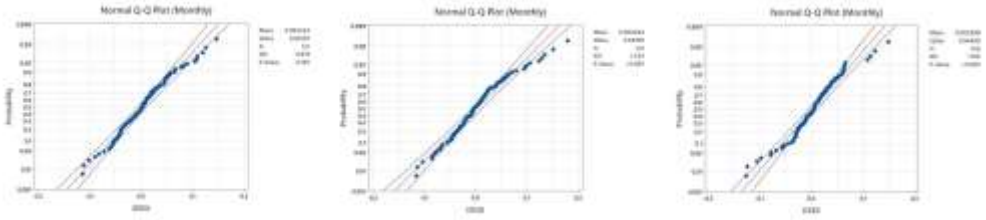

Figure 4 (a-c). Q-Q Plot of DSES Monthly Returns.

The problem of heavy tails has not been observed in the monthly returns in Figure 4 (a-c).
Till now, the paper presented the results of normality graphically. Results of the formal tests of normality are presented next.

Table 4. Results of Normality Tests on Daily Index Returns.

Indices/Tests	Jarque-Bera		Shapiro-Wilk		Anderson-Darling	
	Statistic	P-Value	Statistic	P-Value	Statistic	P-Value
DSEX	13,428.44	0.0000	0.9206	0.0000	26.91	0.0000
DS30	9,619.84	0.0000	0.9222	0.0000	27.91	0.0000
DSES	25,578.48	0.0000	0.9043	0.0000	23.19	0.0000

In case of daily returns, all the tests reject the null hypothesis at the 5% level of significance that the return series follow a normal distribution.

Table 5. Results of Normality Tests on Monthly Index Returns.

Indices/Tests	Jarque-Bera		Shapiro-Wilk		Anderson-Darling	
	Statistic	P-Value	Statistic	P-Value	Statistic	P-Value
DSEX	3.88	0.1431	0.983	0.129	0.619	0.1050
DS30	15.26	0.0004	0.9662	0.0039	1.220	0.0000
DSES	12.33	0.0021	0.9639	0.0048	1.165	0.0000

At a 5% level of significance, monthly returns of DSEX have been found to be normally distributed according to all three tests. However, monthly returns of DS30 and DSES indices have not been found to be normally distributed.
The findings of this paper are similar to some past and recent papers [5-7, 13-17]. It is an indication of reliability of this research.

Conclusions: The normality of stock returns is a topic of debate in asset pricing. While the assumption of normality is often used in finance, empirical evidence presented in this paper suggests that stock returns are not truly normally distributed. Therefore, it is important to use caution when relying on normality assumptions in financial modeling and to consider alternative models that better capture the characteristics of stock returns.
There are many policy implications of non-normality in stock returns. Non-normality can lead to the presence of market anomalies, such as fat tails in the return distribution. These anomalies imply that extreme events occur more frequently than predicted by normal distribution models [25]. Non-normality in stock returns indicates the contra-evidence of random walk behavior of market returns [11]. Overall, understanding the limitations of the normality assumption can help investors and researchers make more accurate and robust financial decisions.
The paper opens up a promising scope for further investigation in several areas. Firstly, extending this research over a longer time horizon could provide valuable insights into the stability of return distributions over time. Secondly, testing the normality of stock returns can be extended to sector-specific or industry-specific stock returns. Finally, comparative studies across different emerging markets or an analysis of cross-asset class returns within Bangladesh could provide a broader perspective on return distribution characteristics. In summary, there exists a rich scope for future research endeavors to build upon the foundation laid by this study and further enhance our comprehension of stock return behavior in the specific context of Bangladesh.

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